

Dr Andrew David Brooke-Taylor

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Major Areas of Interest:

Set Theory: Large Cardinals and Forcing

Applications of Set Theory to Category Theory and Algebraic Topology

Personal:

Born 15 November 1979 in Cairns, Australia.

Australian and British dual citizen.

Education:

University of Vienna: Doctorate in Mathematics, 2007.

Thesis title: Large Cardinals and L -Like Combinatorics

Advisor: Professor Sy Friedman

Employed as a researcher on an Austrian Science Fund grant

Massachusetts Institute of Technology: Work towards Ph.D. in Mathematics, 2001–2005.

Advisors: Professor Haynes Miller, Professor Gerald Sacks

Received Graduate Presidential Fellowship and Teaching Assistantship.

Transferred to the University of Vienna (see above) to complete my doctorate.

The Australian National University: B.Sc.(Hons) in Mathematics, 2000.

Honours thesis title: Derived Categories

Advisor: Professor Amnon Neeman

Received a University Medal.

Postdoctoral Experience:

March 2008 – present: Research Fellow, University of Bristol

September 2007 – March 2008: Teaching Fellow, University of Bristol.

Awards and Honours:

Dean's Educational and Student Advising Award, MIT, 2005.

Invited panelist, *Better Teaching at MIT* series, 2005.

University Medal, The Australian National University, 2000.

Research

Journal articles in print

Large cardinals and definable well-orders on the universe. *Journal of Symbolic Logic* **74**, no. 2 (June 2009), pp 641–654.

I show how one may use Cohen’s method of *forcing* to obtain a model of set theory in which there is a well-order, definable by a formula, on the class of all sets. Moreover, if there are cardinals in the original universe of sets satisfying any of a variety of “large cardinal” properties (corresponding to strong set-theoretic axioms) then those properties can be preserved through the construction for many such cardinals.

(With Sy Friedman, University of Vienna) Large cardinals and gap-1 morasses. *Annals of Pure and Applied Logic* **159** (2009), pp 71–99.

Morasses are intricate combinatorial structures useful in set-theoretic constructions; for example, constructing models of a given theory with a given cardinality where a certain distinguished predicate has another given cardinality. We show how to force to obtain a model of set theory in which there is a morass for every regular uncountable cardinal, and further, a variety of large cardinal properties are preserved.

Articles in preparation

Indestructibility of Vopěnka’s Principle

Vopěnka’s Principle is a large cardinal axiom in set theory that has in recent years found applications to algebraic topology and category theory more generally (see my work with Bagaria and Casacuberta, below). I show that if a universe of sets satisfies Vopěnka’s Principle, then any forcing construction satisfying certain mild conditions will yield a universe which also satisfies Vopěnka’s Principle. In particular, this shows at a stroke that most of the (many) statements that can be shown consistent by forcing are also consistent with Vopěnka’s Principle.

Fraïssé limits for infinite relational languages (with Damiano Testa, Oxford University)

In a departure from my usual field of set theory, this work focuses on finite model theory using Fraïssé limits. We show that for certain sufficiently well-behaved theories over infinite languages, including the theories of hypergraphs, simplicial complexes, and Sperner families, results associated with Fraïssé limits over finite languages still hold. One notable result from this work is the existence of a new zero-one law for first order statements about simplicial complexes.

Forcing Universal Morasses

Universal morasses are morasses (see above) with added structure, encoding the whole power set of the cardinal in question, making them a more powerful tool for constructions. I show that one may obtain a model of set theory containing universal morasses by the method of forcing.

Other lines of research

Applications of large cardinal axioms to category theory and algebraic topology (with Joan Bagaria and Carles Casacuberta, University of Barcelona)

In recent years, large cardinal axioms from set theory have found important applications to category theory, giving a variety of structure theorems for categories, and even solving the long-standing question in algebraic topology of whether Bousfield localisation functors exist for all generalised cohomology theories (in the affirmative: Casacuberta, Scevenels and Smith, *Implications of large-cardinal principles in homotopical localization*, Adv. Math. 197 No. 1, 2005, pp120–139). In our work we aim to develop and extend this interaction between the fields of set theory and category theory. For example, the possibility that one could derive large cardinals consequences from statements such as the Bousfield localisation one above is particularly enticing. I have been invited to Barcelona twice to work in this area with Bagaria and Casacuberta, and we have made some good first steps, reproving category theoretic results in a set theoretic way, and even finding a counter-example to a published result.

Algebraic structure arising from rank-to-rank embeddings (with Sheila Miller, West Point Military Academy)

Some of the strongest known large cardinal axioms have associated algebraic structure that gives rise to free *left self-distributive (LD) monoids*. LD-monoids have close connections with braid groups, and there are certain simple, finitary statements about them that are only known to hold under the above-mentioned strong axioms. In my work with Miller, we have found a new partial order that can be defined on any LD-monoid, and are in the process of investigating this further.

Invited Talks:

Indestructibility of Vopěnka's Principle, University of Leeds Department of Mathematics Logic Seminar, 2009.

Consistency results for Vopěnka's Principle, Young Set Theory Workshop 2009, Barcelona.

Large Cardinals and Definable Well-Orderings of the Universe, Set Theory and its Neighbours 16, University College of London, 2007.

Other Conference Talks:

A new zero-one law for simplicial complexes, 10th Asian Logic Conference, Kobe, 2008.

Forcing to get universal morasses, Workshop on Combinatorial and Descriptive Set Theory, Kyoto, 2008.

Definable Well-Ordering, the GCH, and Large Cardinals, Logic Colloquium 2007, Wroclaw, Poland (slides available on website listed on page 1).

Forcing while preserving I -extendibles, Logic Colloquium 2006, Nijmegen, The Netherlands (slides available on website).

Teaching Experience

Lecturing:

Logic, University of Bristol, first semester 2007–8.

Mathematical Methods for Engineers, MIT, Summer 2003 and Summer 2004.

Linear Algebra, MIT, Summer 2002.

Also, filled in 3 lectures of Axiomatic Set Theory for an unwell colleague, November 2009.

Project Supervision:

Two 4th year undergraduate student projects on “Ramsey-type large cardinal axioms”, 2008–2009; one of my students won the prize for best project that year.

Tutoring:

1st year pure mathematics, University of Bristol, Semester 1, 2007–8.

Linear Algebra, MIT, Fall Semester 2004.

Mathematical writing class, MIT, 2003–2004.

Calculus with Theory, MIT, 2002–2003.

Second year Calculus, James Cook University, Semester 1, 2001.

In-house mathematics tutor, John XXIII College, The Australian National University, 1998–2000.

Class Coordination:

Linear Algebra, MIT, Fall Semester 2004.

Service:

Programme committee chair, Young Set Theory Workshop 2010.

Refereeing for *Mathematical Logic Quarterly*.

Recent Grants:

Travel grant to give a talk at the Indian School in Logic and its Applications, January 2010, from the organisers.

Exchange Visit Grant to visit Barcelona for the month of October 2009 for joint work, European Science Foundation “InftyNet” programme.

Short Visit Grant to visit Barcelona for 10 days in June 2009 for joint work, European Science Foundation “InftyNet” programme.

Travel grant to attend the Infinite Combinatorics Without the Axiom of Choice workshop, Bonn, June 2009, from the organisers.

Travel grant to attend the 10th Asian Logic Conference in Kobe and give a contributed talk, September 2008, from the organisers.